Eidgenössisches Amt für Messwesen
Office fédéral de métrologie
Ufficio federale di metrologia
Swiss Federal Office of Metrology

CCL Key Comparison

CCL-K1

Calibration of gauge blocks by interferometry

Final report

Wabern, November 2000 / R. Thalmann

Content

1 Introduction

The metrological equivalence of national measurement standards and of calibration certificates issued by national metrology institutes is established by a set of key comparisons chosen and organized by the Consultative Committees of the CIPM or by the regional metrology organizations in collaboration with the Consultative Committees.

At its meeting in September 1997, the Consultative Committee for Length, CCL, has decided upon a key comparison on gauge block measurements by interferometry, named CCL-K1, starting in spring 1998, with the Swiss Federal Office of Metrology (OFMET) as the pilot laboratory.

The results of this international comparison contribute to the mutual recognition arrangement (MRA) between the national metrology institutes of the Metre Convention. This CIPM key comparison is combined - where necessary - with regional comparisons (RMO key comparisons) following exactly the same scheme. Laboratories participating in both, the CIPM and the RMO comparisons establish the link between these comparisons and assure their equivalence.

2 Organisation

According to the rules set up by the BIPM [1] a small group from the list of participating laboratories has drafted the detailed technical protocols. The group was composed of Jennifer Decker from the NRC, Canada, Nicholas Brown from CSIRO/NML, Australia and the pilot laboratory, all experienced in organizing gauge block comparisons and representing also a regional metrology organisation.

2.1 Participants

Laboratory Code	Contact person, Laboratory	Phone, Fax, email
NPL	Andrew Lewis NPL Dimensional Metrology Teddington Middlesex TW11 OLW England	Tel. +44 208 943 61 24 Fax +44 208 943 29 45 e-mail: Andrew.Lewis@npl.co.uk
LNE	Georges Vailleau BNM-LNE Laboratoire National d'Essais 1, rue Gaston Boissier F-75015 Paris France	Tel. +33 1 40 43 3777 Fax +33 1 40 43 3737 e-mail: Georges Vailleau@lne.fr
NRC	Jennifer Decker Institute for National Measurement Standards National Research Council Canada Ottawa, K1A 0R6 Canada	Tel. +1 613 991 1633 Fax +1 613 952 1394 e-mail: Jennifer.Decker@nrc.ca
NIST	John R. Stoup National Institute of Standards and Technology NIST Room B113, Metrology Building Gaithersburg, MD 20899-0001 USA	Tel. +1 301 975 3476 Fax +1 301 869 0822 e-mail: John.Stoup@nist.gov

CENAM	Miguel Viliesid CENAM Metrologia Dimensional Apartado Postal 1-100 Centro 76000 Queretaro, Qro Mexico	Tel. +52 42 11 0574 Fax +52 42 11 0577 e-mail: mviliesi@cenam.mx
VNIIM	Larissa Yu. Abramova Alexander N. Korolev VNIIM 19, perspective de Moskovsky 198005 St. Petersburg Russia	Tel. +7 812 251 8638 Fax + 812 113 0114 e-mail: al_korolev@mail.ru
CSIRO	Nicholas Brown CSIRO/NML Division of Telecommunications and Industrial Physics PO Box 218 Lindfield, NSW 2070 Australia	Tel. +61 2 9413 7157 Fax +61 2 9413 7474 email: nick.brown@tip.csiro.au
NRLM	Katuo Seta National Research Laboratory of Metrology NRLM 1-1-4 Umezono Tsukuba, Ibaraki 305-8563 Japan	Tel. +81 298 54 4030 Fax +81 298 50 1456 e-mail: seta@nrlm.go.jp
KRISS	Chu-shik Kang Korea Research Institute of Standards and Science KRISS P.O. Box 102, Yusong Taejon 305-600 Korea	Tel. +82 42 868 5103 Fax +82 42 868 5608 e-mail: cskang@kriss.re.kr
NIM	Shen Shaoxi National Institute of Metrology NIM No. 18, Bei San Huan Dong Lu Beijing 100013 China	Tel. +86 10 6422 6657 Fax +86 10 6421 8703 e-mail: shenshaoxi@ihw.com.cn
Coordinator:		
OFMET	Ruedi Thalmann OFMET Swiss Federal Office of Metrology Lindenweg 50 CH-3003 Bern-Wabern Switzerland	Tel. +41 31 323 33 85 Fax +41 31 323 3210 e-mail: rudolf.thalmann@eam.admin.ch

Table 1. Participating laboratories.

2.2 Schedule

The comparison has been carried out in a mixed form, circulation and star-type. After the standards were circulated in a region, they were sent back to the pilot laboratory before being circulated in the next region. Each major region had a local coordinator who helped solve the regional transportation and customs problems and helped maintain the tight time schedule.

RMO	Labora-tory	Original schedule	Date of measurement	Results received
EUROMET	OFMET	March 1998	March 1998	
	NPL	May 1998	May 1998	June 1998
	LNE	June 1998	June 1998	July 1999
Pilot Lab	OFMET	July 1998	July 1998	
NORAMET	NRC	August 1998	August 1998	September 1998
	NIST	September 1998	September 1998	January 1999
	CENAM	October 1998	October 1998	January 1999
Pilot Lab	OFMET	November 1998	December 1998	
COOMET	VNIIM	January 1999	January 1999	February 1999
Pilot Lab	OFMET	February 1999	February 1999	
APMP	CSIRO	March 1999	March 1999	April 1999
	NRLM	April 1999	April 1999	June 1999
	KRISS	May 1999	May 1999	September 1999
	NIM	June 1999	July 1999	August 1999
Pilot Lab	OFMET	August 1999	September 1999	

Table 2. Time schedule of the comparison.

3 Standards

10 gauge blocks of steel and 10 gauge blocks of tungsten carbide were circulated. The gauge blocks were of rectangular cross section, according to the international standard ISO 3650. The thermal expansion coefficient of the gauge blocks was measured by the pilot laboratory before the comparison. These results were given to the participating laboratories in the technical protocols.

The tungsten carbide gauge block marked as 90 mm was 1.39145 mm shorter than its nominal length [2]. It was manufactured to have similar excess fringe fractions as a 90 mm gauge block for all laser and spectral lamp wavelengths usually used in gauge block interferometry. Only the laboratories making a predetermination of the length by mechanical comparison could become aware of this fact. The purpose of this rogue gauge was to draw attention to the need for mechanical predetermination of the length when no additional information about the length is available other than the nominal length. On the other hand, gauge blocks with such a large deviation from nominal length do not exist in practice, in particular with a length having the same excess fringe fractions. Also, the time for measurement allocated to the participants did not always allow for a mechanical comparison, in particular where this would usually be done at another place. The results for this gauge block do therefore not necessarily represent the measurement capability of the laboratories and are not reported here.

Steel gauge blocks:

Identification	Nominal length (mm)	Expansion coeff. (10^{-6} K^{-1})	Manufacturer
2’10282	0.5	11.52 ± 0.1	CARY
3’23288	1.01	11.52 ± 0.1	CARY
21’23584	1.1	11.52 ± 0.1	CARY
1’0071	6	11.52 ± 0.1	CARY
16’0087	7	11.52 ± 0.1	CARY
7’0103	8	11.52 ± 0.1	CARY
18’23395	15	11.52 ± 0.1	CARY
24’23259	80	11.56 ± 0.03	CARY
7’23260	90	11.72 ± 0.03	CARY
29’23539	100	11.52 ± 0.03	CARY

Tungsten carbide gauge blocks:

Identification	Nominal length (mm)	Expansion coeff. (10^{-6} K^{-1})	Manufacturer
9’2605	0.5	4.24 ± 0.1	CARY
20’23289	1	4.24 ± 0.1	CARY
10’20632	1.01	4.24 ± 0.1	CARY
20’20987	1.1	4.24 ± 0.1	CARY
2’22685	6	4.24 ± 0.1	CARY
24’95598	7	4.24 ± 0.1	CARY
19’22087	8	4.24 ± 0.1	CARY
B 32364	80	4.26 ± 0.02	Select
H 580223-007	90	4.25 ± 0.02	-
B 32365	100	4.27 ± 0.02	Select

Table 3. Standards used in the comparison. The uncertainties for the thermal expansion coefficient are given as standard uncertainties.

4 Measurement instructions and reporting

Before calibration, the gauge blocks had to be inspected for damage to the measurement surfaces. Any scratches, rusty spots or other damage had to be documented by a drawing using forms appended to the instructions.

The measurement quantity was the central length of the gauge blocks, as defined in the International Standard ISO 3650. The gauge blocks had to be measured by interferometry, in their vertical position while wrung to a flat plate. The measurement result to be reported was the deviation of central length from nominal length, $Dl = l - L$. The results of the measurements on both sides (Dl_{left} and Dl_{right}) by wringing each measurement face in turn to the reference flat and the average of the two wringings had to be reported. The measurement results had to be corrected to the reference temperature of 20°C using the thermal expansion coefficients given above. Additional corrections (aperture, phase correction) had to be applied according to the usual procedure of the laboratory.

The uncertainty of measurement had to be estimated according to the *ISO Guide for the Expression of Uncertainty in Measurement*. In order to achieve optimum comparability, a mathematical model [3] containing the principal influence parameters for gauge block calibration by interferometry was given in the technical protocol.

5 Measurement methods and instruments used by the participants

All laboratories have measured the gauge blocks by optical interferometry, applying the method of fringe fractions. The instruments used and further details to the measurement procedures and conditions are summarized in table A1 in the appendix.

Most of the laboratories applied phase corrections to compensate for any difference in material and surface roughness between the gauge blocks and the platens used for wringing the gauge blocks. This correction is specified in the techical protocol and is also required by the ISO 3650 standard. Table 4 gives the material of the reference flats used by the laboratories and the range of the applied corrections, representing the phase difference values between the platens and the gauge blocks. It has to be noted, that these numbers may be different as they indicate differences between the material and the surface finish of the various platens used by the laboratories.

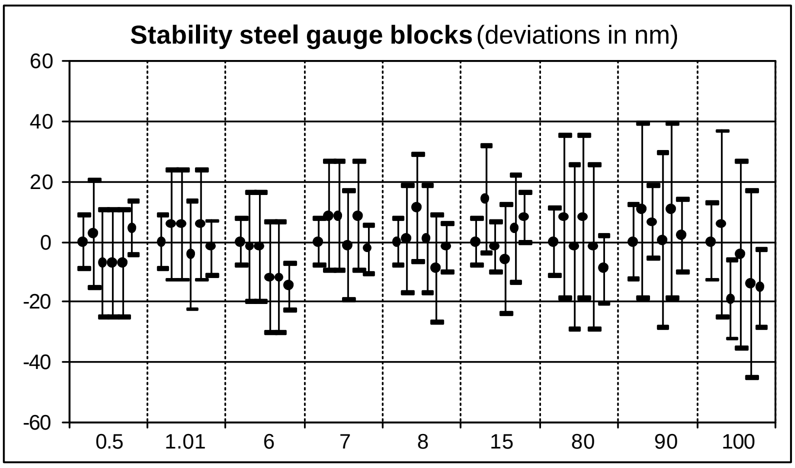
	Steel gauge blocks		**TC gauge blocks**	
	Material	**Range / nm**	**Material**	**Range / nm**
OFMET	steel	-18	TC	-15
NPL	steel	-37 .. -41	TC	-5 .. -9.7
LNE	steel	-10 .. -20	steel	-35
NRC	fused silica	55	fused silica	39 .. 44
NIST	steel	7	steel	-10
CENAM	steel	-33	steel	-50
VNIIM	steel	0	quartz	0
CSIRO	steel	-6	steel	-18
NRLM	glass	20 .. 40	glass	4 .. 20
KRISS	steel	-5.1 .. 0.3	TC	1.5 .. 5.9
NIM	steel	0	TC	0

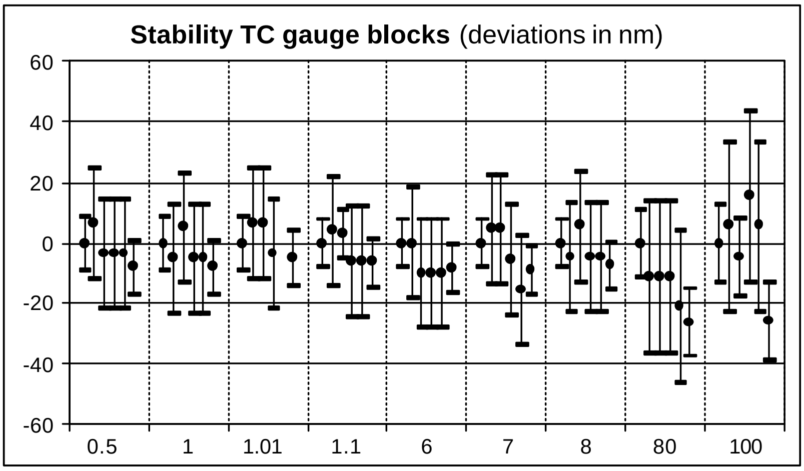
Table 4. Material of the reference flats to wring the gauges and range of phase correction applied to the measurement results for the steel and tungsten carbide (TC) gauge blocks, respectively.

Two laboratories (VNIIM and NIM) did not determine any phase correction, but assumed a zero value. It must be noted, however, that in three of these four cases the same material has been used for the reference platen as the gauge blocks, therefore small correction values could be expected. This is reflected in the fact that the average deviations of NIM for both steel and tungsten carbide, and of VNIIM for steel, were reasonably small. Only the tungsten carbide results of VNIIM were offset by more than 30 nm from the average of the others, which is obvious, since quartz platens were used.

6 Stability and condition of the gauge blocks

The stability of the gauge blocks was monitored by different calibrations performed by the pilot laboratory (OFMET) during the comparison. An interferometric calibration was made at the beginning (March 1998) and at the end (September 1999). Five calibrations by mechanical comparison were made between the regional loops and at the end.Figures 1 and 2 show the results of these calibrations for the steel and the tungsten carbide gauge blocks, respectively. All results are given as the difference with respect to the initial calibration. The uncertainty bars are standard uncertainties. It has to be noted that for monitoring the stability, the uncertainties would be less as common factors can be ignored (such as the phase correction in interferometry and the reference standards for mechanical comparison). The figures show, that no significant change in length can be observed.





Figures 1 and 2. Stability measurements of the gauge blocks during the comparison (uncertainty bars are standard uncertainties).

The gauge blocks were essentially free of any damage at the beginning of the comparison. The participating laboratories were asked to document any scratches and other damage to the measurement surfaces with a drawing, made when receiving the gauge blocks. The surface quality of the tungsten carbide gauge blocks remained impeccable until the end of the comparison. The steel gauge blocks, however, got more and more scratches from the repeated wringing and also some small spots could be observed, probably caused by rust. This degradation of the surface quality must be considered to be normal in the course of a comparison. All except the 1.1 mm steel gauge block wrung reasonably well at the final calibration. The 1.1 mm gauge block could not be calibrated by the last laboratories and therefore the results are not reported here. When arriving at CSIRO the 100 mm steel gauge block showed three small edge scratches which were slightly high. Since the burs risked damage to the platen, they were carefully repaired by CSIRO using a granite dressing plate.

The temperature of the standards has been monitored by a small data logger which travelled in the transportation container during the whole comparison. An evaluation of all the data showed that the temperature remained between 5 °C and 35 °C. Since it can be assumed that the gauge blocks were never removed from the transportation container except in a laboratory environment, it can be concluded that the gauge block temperature was kept in the same range. Figure 3 shows the recording of the last 6 ½ months.

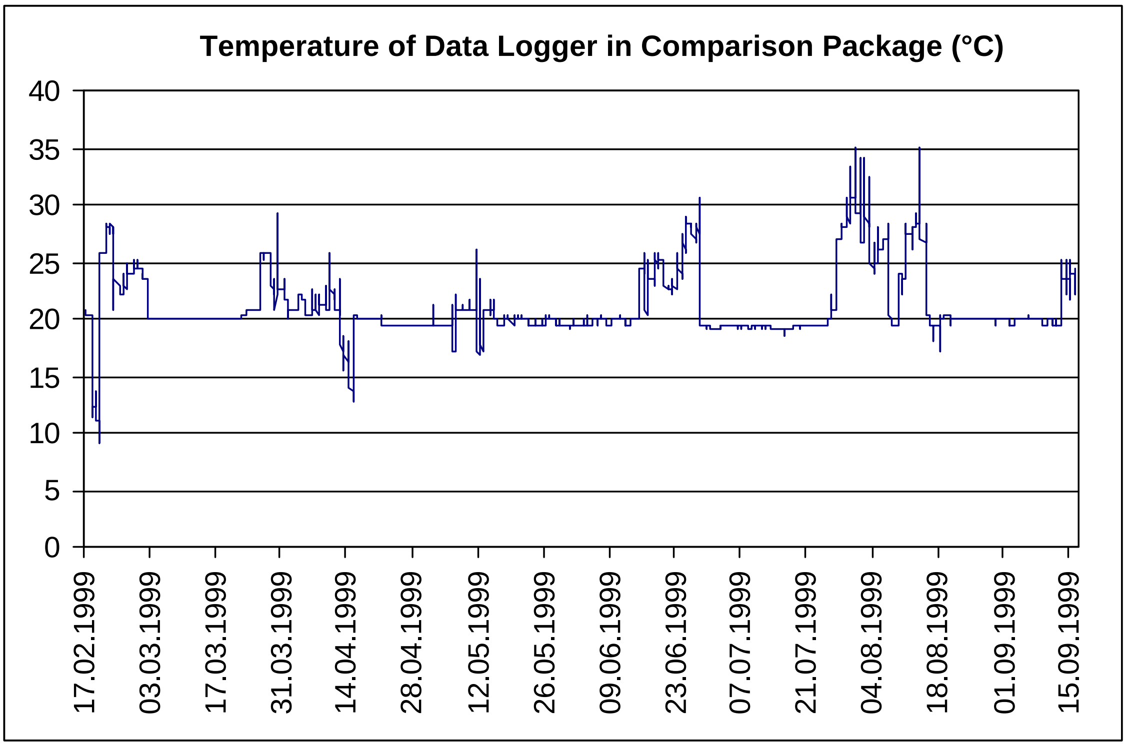


Figure 3. Temperature recording during the second half of the comparison of the data logger accompanying the transportation container.

7 Measurement results

7.1 Deviation from nominal length, steel gauge blocks

All measurement results for the deviation from nominal length for 9 steel gauge blocks are represented in table 5(a). Table 5(b) shows the corresponding combined standard uncertainties as they were reported by the participating laboratories.

	Gauge block nominal length / mm								
Lab	**0.5**	**1.01**	**6**	**7**	**8**	**15**	**80**	**90**	**100**
OFMET	17	34	52	31	-1	16	22	-21	-96
NPL	20	25.5	54.5	33.5	1.5	22.5	38.5	-14	-140
LNE	15	25	54	35	4	20	28	-24	-110
NRC	29	28	36	30	2	14	9	-37	-126
NIST	26	42	57	34	9	30	33	-23	-117
CENAM	15	20	47	26	-3	13	21	-19	-119
VNIIM	*	60	68	25	32	36	25	-32	-104
CSIRO	28	46	53	37	12	51	27	-20	-114
NRLM	23.9	17.7	44.1	27	-2.2	15.1	47.3	9.1	-89.4
KRISS	18.7	20.3	22.1	12.8	-24.2	8.1	30.4	-18.4	-104.3
NIM	30	48	56	42	12	28	44	18	-90

Table 5a. Deviation from nominal length (in nm) of the steel gauge blocks, as reported by the laboratories. (: not measured due to flatness out of tolerance).*

	Gauge block nominal length / mm								
Lab	**0.5**	**1.01**	**6**	**7**	**8**	**15**	**80**	**90**	**100**
OFMET	9	9	8	8	8	8	11	12	13
NPL	14	14	14	14	14	15	28	31	33
LNE	10	10	10	10	10	10	14	15	16
NRC	13	13	14	14	14	14	21	22	24
NIST	8.9	9	9.4	9.5	9.6	10.3	16.1	17	17.9
CENAM	7	7	7.1	7.1	7.2	7.4	15.6	17.3	18.7
VNIIM		8	8	8	8	8	12	14	15
CSIRO	9	9	9	9	9	9	14	15	16
NRLM	8.6	10.3	10.3	8.7	10.3	10.9	13.5	14.3	16.3
KRISS	13.1	12.2	13.6	11	11	13.2	17	18.9	20.6
NIM	5.4	5.4	5.5	5.5	5.5	5.6	8.9	9.6	10.3

Table 5b. Combined standard uncertainties (in nm) for the measurement of the steel gauge blocks, as reported by the laboratories.

The figures 4 (a-i) show the results with error bars corresponding to the standard uncertainty.

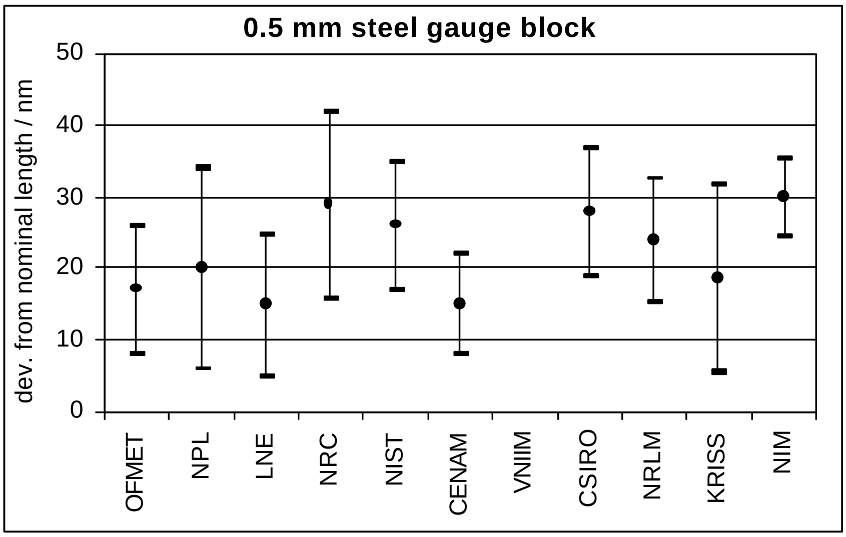


Figure 4a

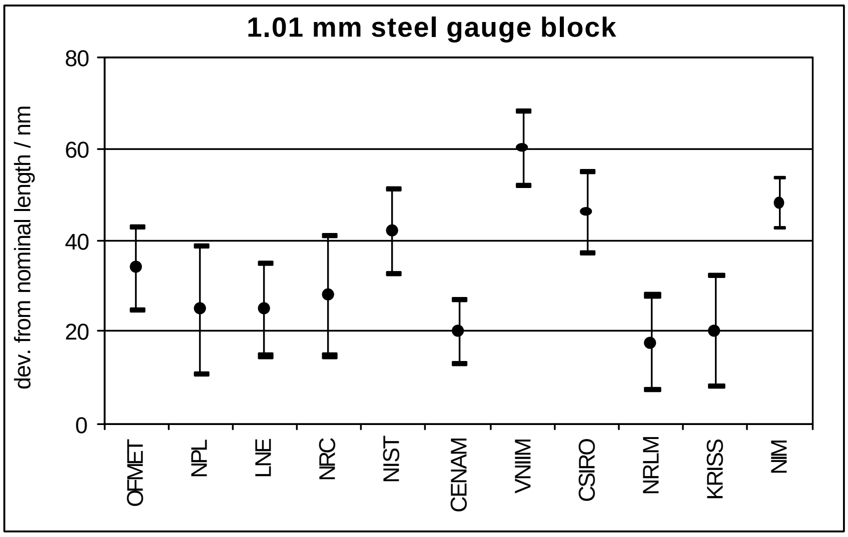


Figure 4b

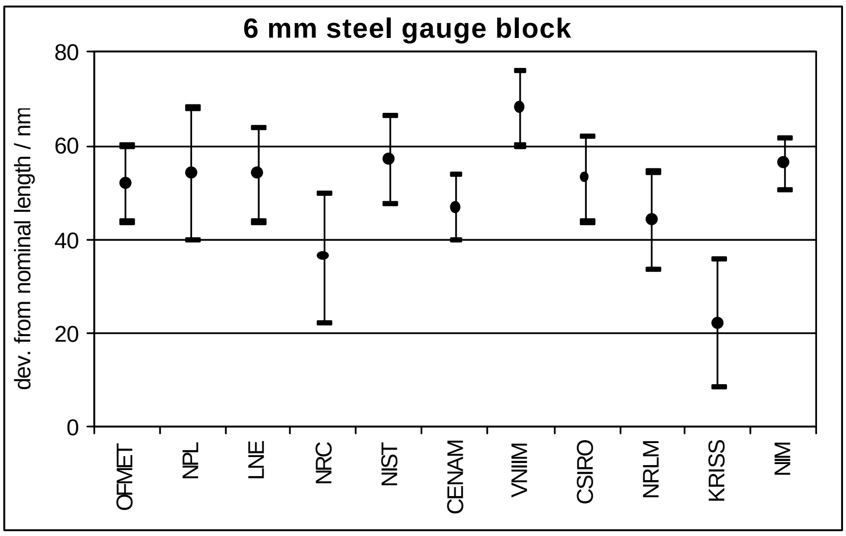


Figure 4c

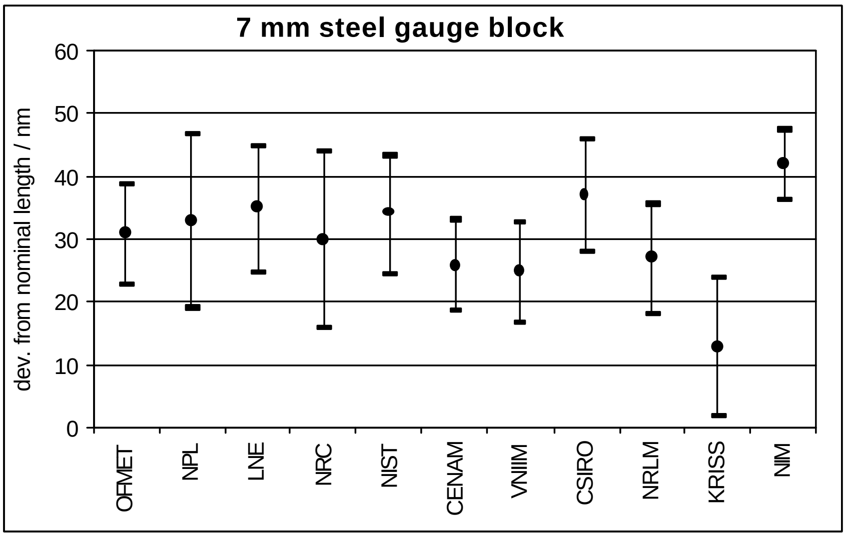


Figure 4d

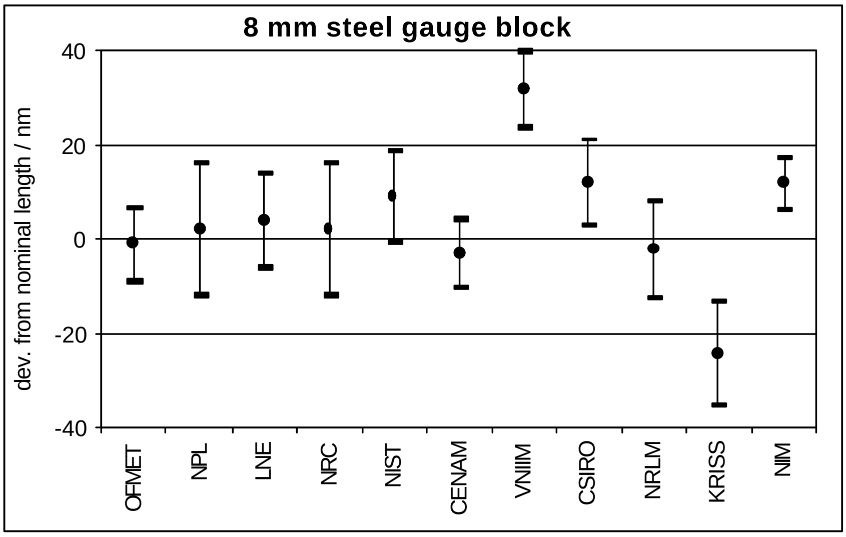


Figure 4e

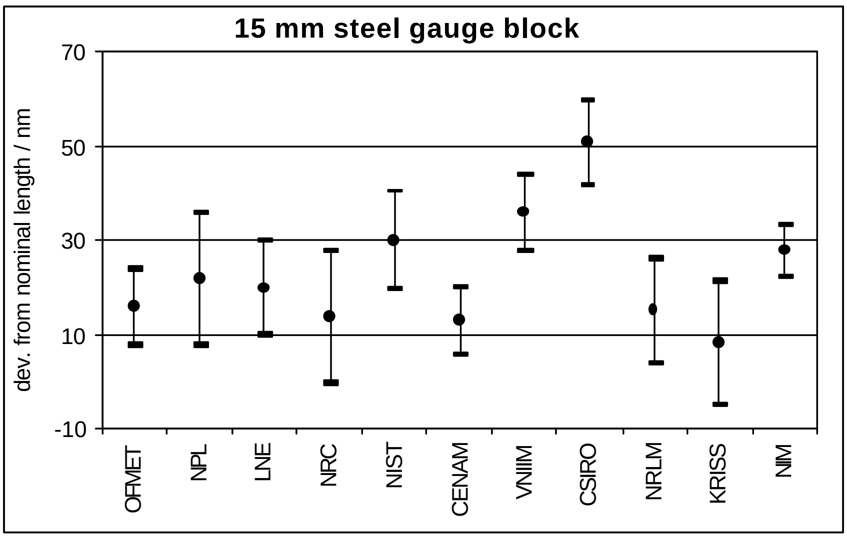


Figure 4f

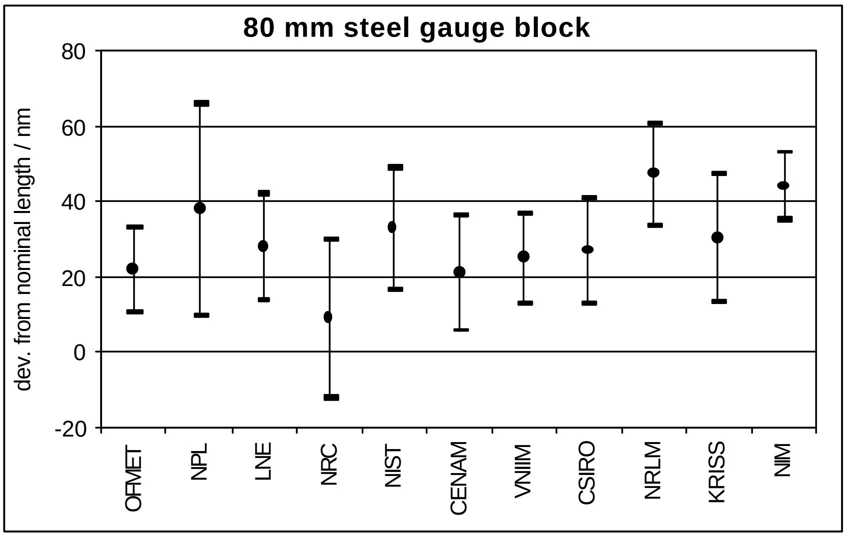


Figure 4g

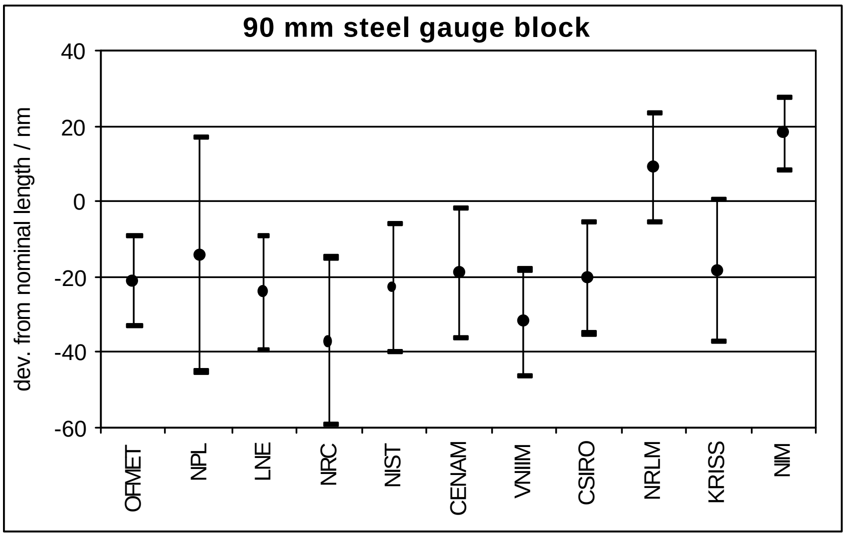


Figure 4h

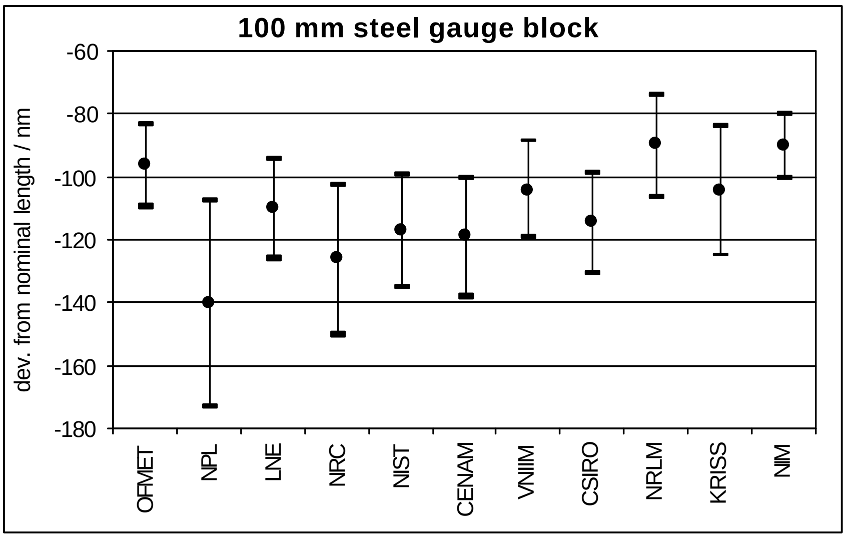


Figure 4i

7.2 Deviation from nominal length, tungsten carbide gauge blocks

All measurement results for the deviation from nominal length for 9 tungsten carbide gauge blocks are represented in table 6(a). Table 6(b) shows the corresponding combined standard uncertainties as they were reported by the participating laboratories.

	Gauge block nominal length / mm								
Lab	**0.5**	**1**	**1.01**	**1.1**	**6**	**7**	**8**	**80**	**100**
OFMET	23	15	24	-54	-50	26	45	111	-66
NPL	24	15	23	-51	-48	29	53.5	116.5	-94
LNE	41	30	37	-36	-34	44	56	116	-65
NRC	29	18	29	-51	-48	28	49	90	-97
NIST	35	24	32	-38	-41	39	58	101	-78
CENAM	-2	-9	10	-72	-61	17	35	87	-101
VNIIM	24	-9	-12	-82	-96	-6	30	64	-110
CSIRO	31	33	39	-32	-29	49	64	105	-71
NRLM	18.8	12.5	35.1	-66.4	-50.4	13.2	40.5	101.2	-69.6
KRISS	17.8	8.8	19.6	-62	-57.9	27	37.3	112.3	-67.5
NIM	32	21	28	-50	-48	32	58	113	-58

Table 6a. Deviation from nominal length (in nm) of the tungsten carbide gauge blocks, as reported by the laboratories.

	Gauge block nominal length / mm								
Lab	**0.5**	**1**	**1.01**	**1.1**	**6**	**7**	**8**	**80**	**100**
OFMET	9	9	9	9	8	8	8	10	11
NPL	14	14	14	14	14	14	14	17	19
LNE	10	10	10	10	10	10	10	11	11
NRC	13	13	13	13	14	14	14	21	24
NIST	8.9	9	9	9	9.4	9.5	9.6	16.1	17.9
CENAM	7	7	7	7	7	7	7	9.4	10.4
VNIIM	8	8	8	8	8	8	8	12	14
CSIRO	9	9	9	9	9	9	9	11	12
NRLM	8.6	8.6	8.6	10.3	8.6	8.6	8.6	9.8	10.4
KRISS	9.9	10	9.7	9.4	10	10.4	10.6	14.6	15.8
NIM	5.4	5.4	5.4	5.4	5.5	5.5	5.5	7.2	8

Table 6b. Combined standard uncertainties (in nm) for the measurement of the tungsten carbide gauge blocks, as reported by the laboratories.

The figures 5 (a-i) show the results with error bars corresponding to the standard uncertainty.

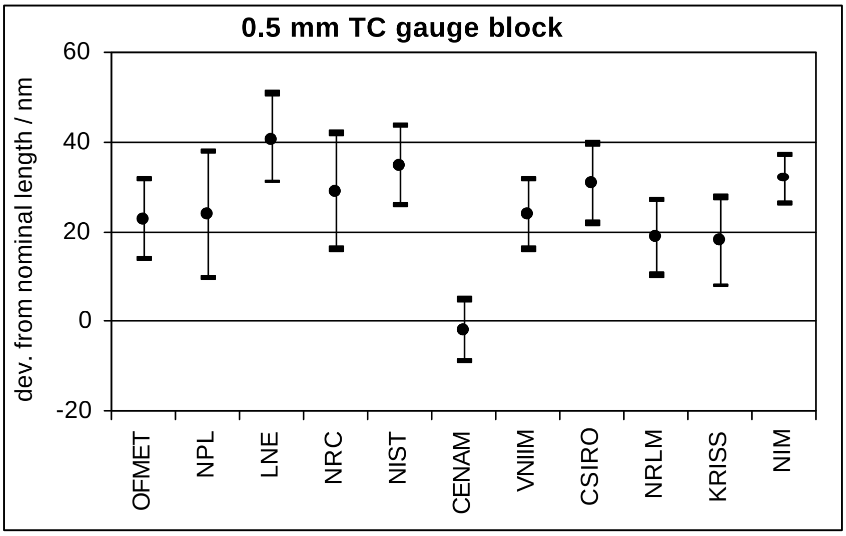


Figure 5a

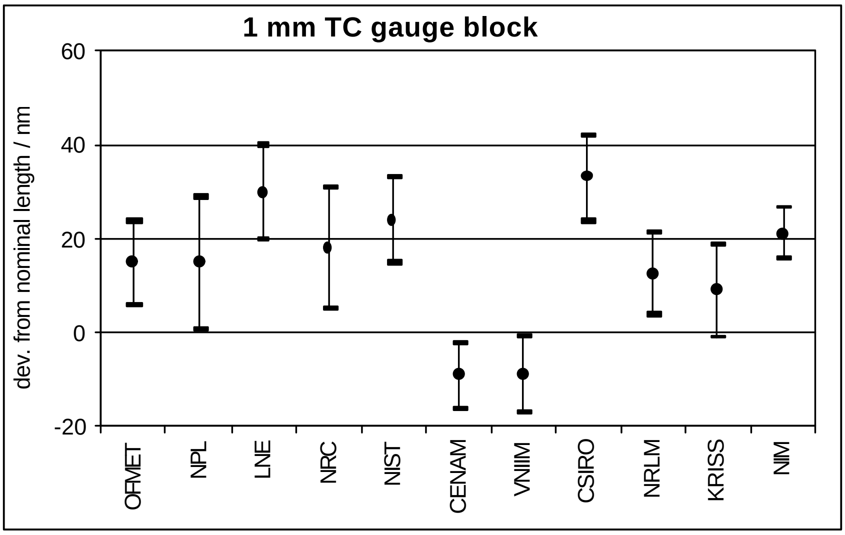


Figure 5b

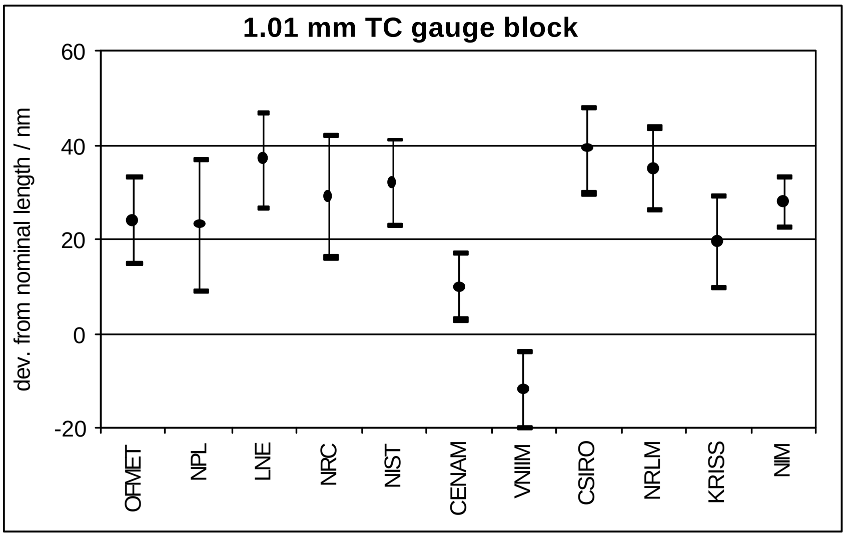


Figure 5c

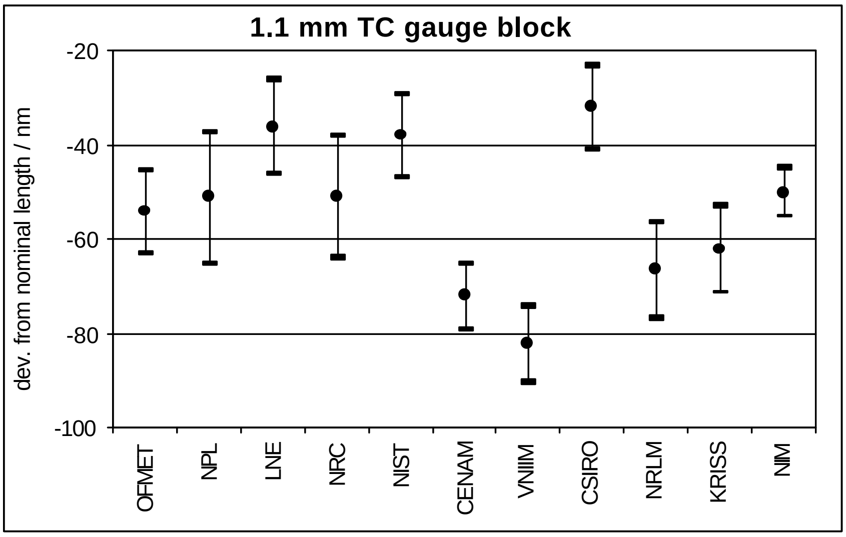


Figure 5d

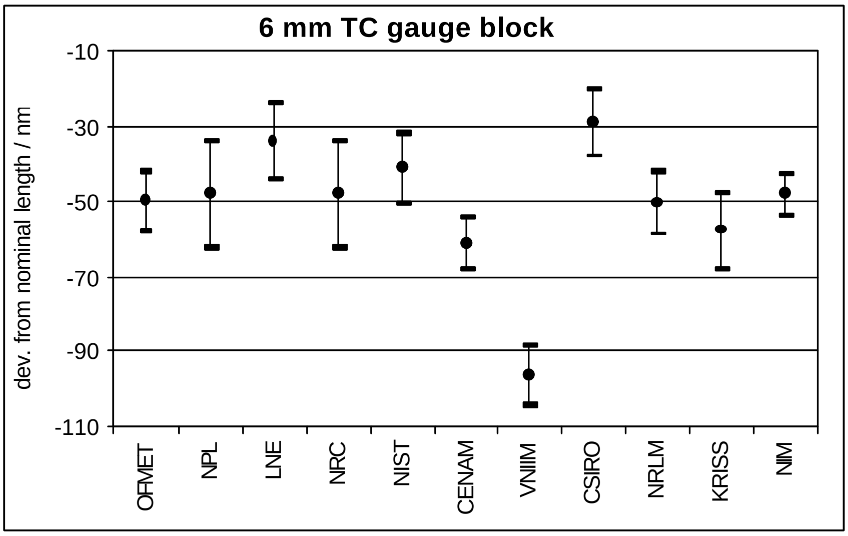


Figure 5e

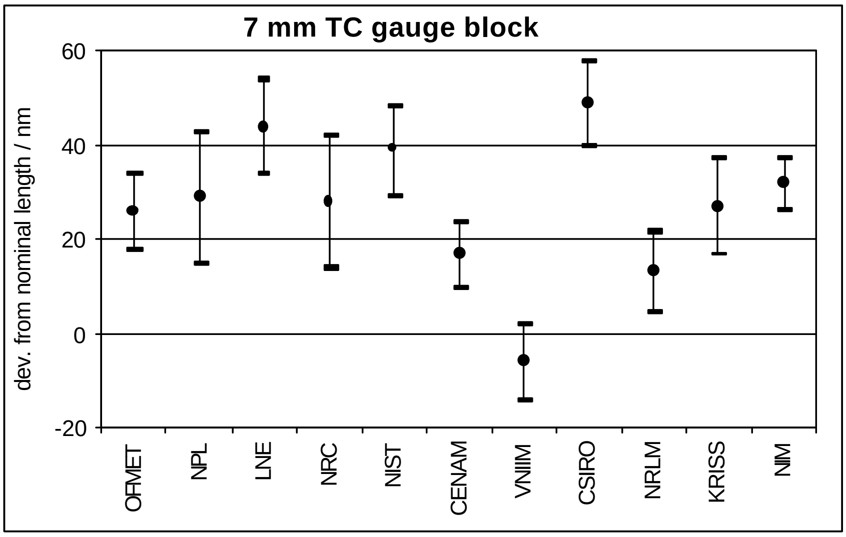


Figure 5f

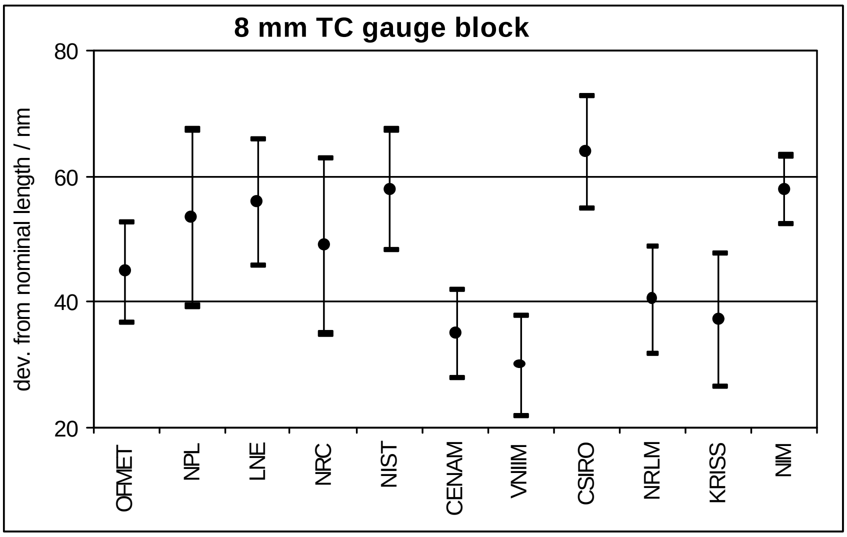


Figure 5g

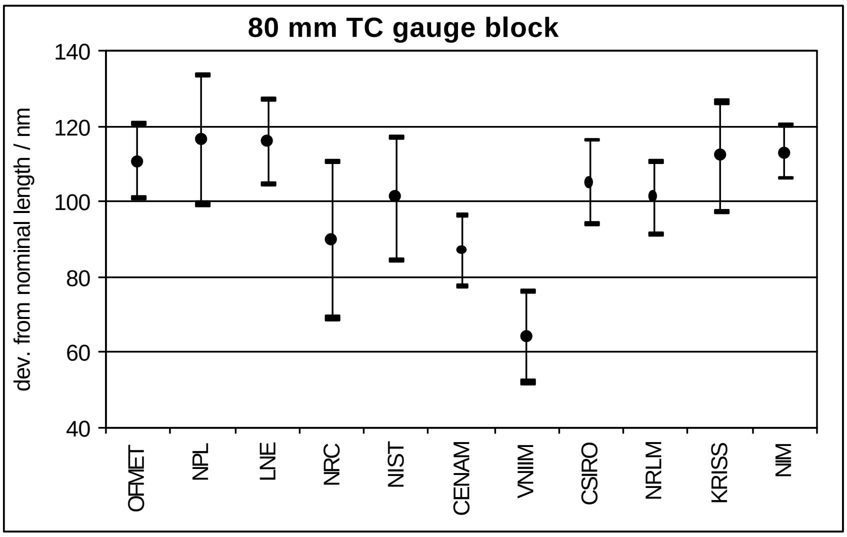


Figure 5h

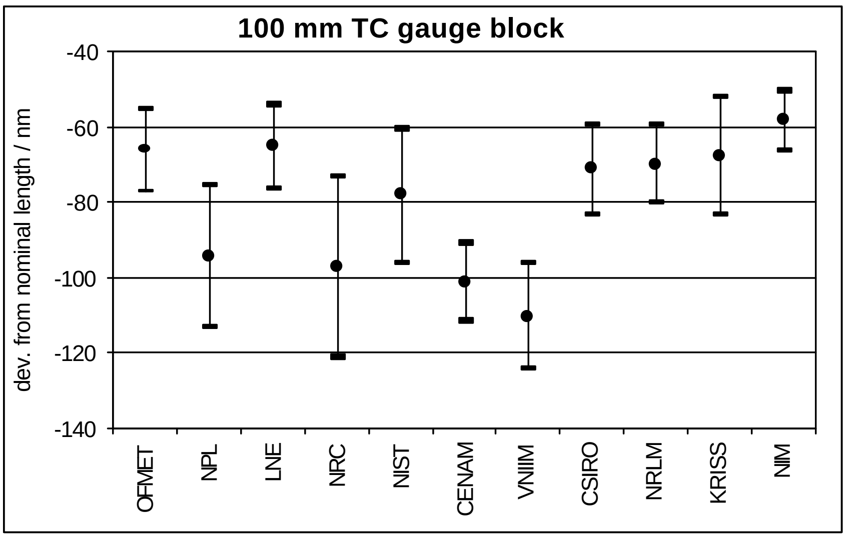


Figure 5i

7.3 Difference between left and right wringing

The laboratories had to measure the gauge blocks wrung to both the left and the right measurement surface and to report both these results and the mean. Table 7 shows the standard deviation and the absolute maximum value of the differences between the two wrings, for all laboratories for both materials, steel and tungsten carbide. These figures might be interpreted as the ability of the laboratories to repeatedly wring gauge blocks.

steel gauge blocks

	OFMET	NPL	LNE	NRC	NIST	CENAM	VNIIM	CSIRO	NRLM	KRISS	NIM
Stdev	4	7	4	12	7	6	3	8	10	9	7
Max	10	13	7	26	24	13	7	14	23	15	19

tungsten carbide gauge blocks

	OFMET	NPL	LNE	NRC	NIST	CENAM	VNIIM	CSIRO	NRLM	KRISS	NIM
Stdev	5	10	5	7	6	7	2	6	8	5	9
Max	10	22	10	13	11	13	4	13	16	11	20

Table 7. Standard deviation and absolute maximum value of the differences between left and right wringing for both materials, steel and tungsten carbide.

Figures 6 (a) and (b) show the differences between left and right wringing for all gauge blocks and all laboratories.

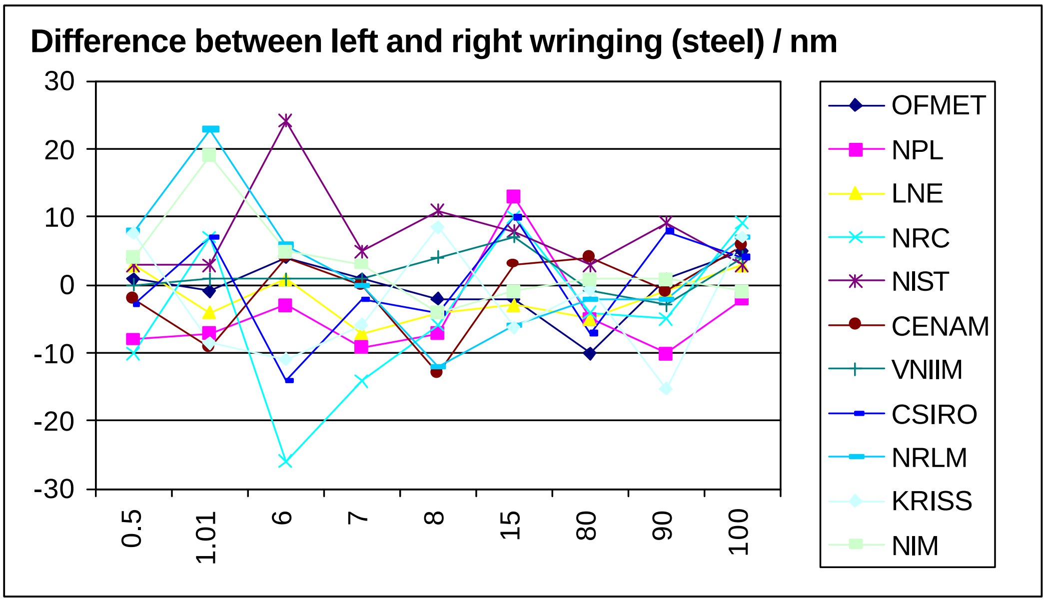


Figure 6a. Differences between left and right wringing for all steel gauge blocks and all laboratories.

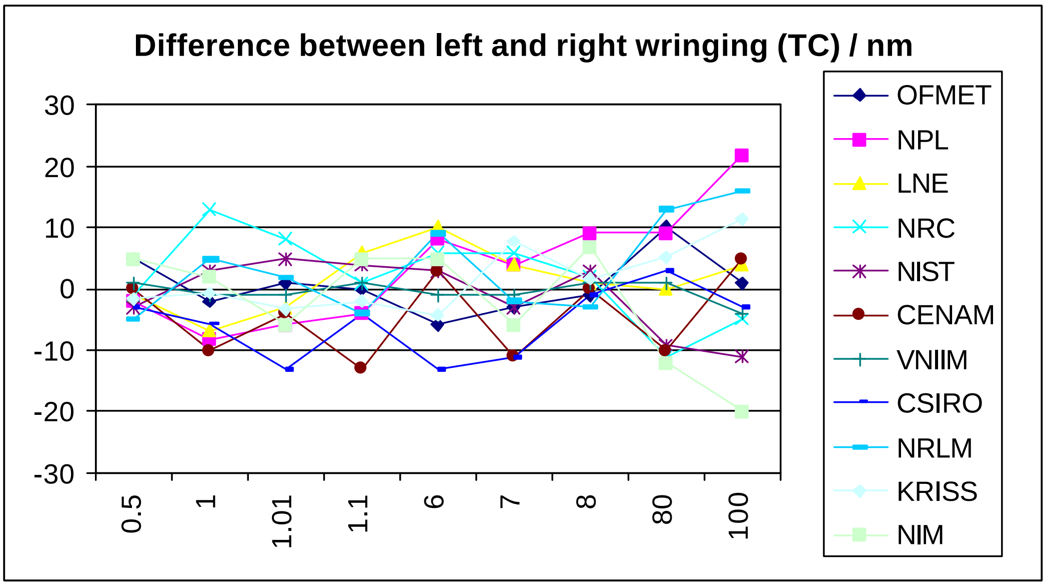


Figure 6b. Differences between left and right wringing for all tungsten carbide gauge blocks and all laboratories.

8 Measurement uncertainties

The participants were asked in the technical protocols of the comparison to estimate the uncertainty of measurement according to the *ISO Guide for the Expression of Uncertainty in Measurement*. An example mathematical model [3] was given and the participants were encouraged to use this model as closely as possible which allows for a detailed comparison of all the uncertainties. Thus all laboratories have indicated their uncertainty contributions according to the various components of that model. Some laboratories indicated, however, that they usually use another model for the uncertainty leading to somewhat different results.

The following contributions to the combined uncertainty were taken into account:

l_i vacuum wavelengths of the different light sources used;

F_i fractional part of fringe order;

n index of refraction of the air;

Dt_G difference of the gauge block temperature from the reference temperature of 20 °C;

a linear coefficient of thermal expansion of the gauge block;

dl_W obliquity correction for the shift in phase resulting from the angular alignment errors of the collimating assembly;

Dl_s aperture correction accounting for the shift in phase resulting from the finite aperture diameter s of the light source;

dl_A correction for wave front errors as a result of imperfect interferometer optics;

dl_G correction accounting for flatness deviation and variation in length of the gauge block;

dl_w length attributed to the wringing film;

Dl_F phase change accounting for the difference in the apparent optical length to the mechanical length;

Dl_{Ff} variation of the phase change correction within the group of gauge blocks. Note that this term was not mentioned in the model given to the participants [3] and was therefore most often not applied. In some cases, however, a variation of 20 nm within one set of gauge blocks has been observed (private communication of NPL).

In table 8, the uncertainty contributions are summarized for all laboratories. The numerical values are standard uncertainties given in nanometers for the case of a steel gauge block. The length dependent terms are written in italics and were calculated for a gauge block length of 100 mm. In the last row, the combined standard uncertainty has been calculated by a simple quadratic sum. This may not necessarily be identical to the combined uncertainty quoted by the laboratory for the 100 mm steel gauge blocks, because they might have used further contributions, correlations and second order terms, which are not given in the table.

	OFMET	**NPL**	**LNE**	**NRC**	**NIST**	**CENAM**	**VNIIM**	**CSIRO**	**NRLM**	**KRISS**	**NIM**
l_i	*2.0*	*1.0*	*0.3*	*0.5*	*0.3*	*3.0*	*0.4*	*0.1*	*0*	*2.5*	*1.2*
F_i	3.2	1.0	4.2	2.0	4.5	3.0	4.2	2.6	4.1	1.3	0.6
n	*5.8*	*4.2*	*3.5*	*20.0*	*3.0*	*2.5*	*2.0*	*7.0*	*3.7*	*1.9*	*4.8*
Dt_G	*7.4*	*29.6*	*11.5*	*7.2*	*8.5*	*16.7*	*11.0*	*10.0*	*9.4*	*11.2*	*6.9*
a	*0.3*	*0.3*	*0*	*0.1*	*0.8*	*2.5*	*10.0*	*6.0*	*3.0*	*2.4*	*0.6*
dl_W	*0.6*	*0.6*	*0*	*0.8*	*0.1*	*0.6*	*1.0*	*1.0*		*0.8*	*0.1*
Dl_s	*0.5*	*0.9*		*0.2*		*0.4*	*0.2*	*0.2*	*0.4*	*1.1*	*2.0*
dl_A	3.7	5.2	3.0	3.0	3.0	3.5	3.0	4.0	2.9	3.5	3.0
dl_G	1.4	3.5	5.0	2.0		1.4	3.0	2.9	2.5	6.5	2.7
dl_w	3.5	5.0	5.0	8.0	4.0	2.9	5.0	5.7	6.9	4.8	3.6
Dl_F	4.2	5.0	3.4	10.0	5.8	4.0	2.0	5.0	8.1	1.5	
Dl_{Ff}	3.5	10.0									
$\boldsymbol{u_c}$	**12.7**	**33.0**	**15.3**	**25.2**	**12.7**	**18.7**	**17.1**	**16.6**	**16.0**	**15.0**	**10.3**

Table 8. Standard uncertainties (in nm) quoted by the different laboratories for the uncertainty contributions given in the model of the technical protocols, and combined uncertainty calculated therefrom.

9 Statistical analysis of the results

The reported measurement results are analysed by simple statistical means. This allowed the identification of any significant bias or outliers, and the investigation of the statistical distribution of the results.

9.1 Average deviation and standard deviation

For each laboratory, the average <Δ*l*> of the deviations from the mean over all gauge blocks has been calculated:

$$< \Delta l >= \frac{1}{N}\sum_{j=1}^{N}(x_j - \overline{x}_j) \qquad (1)$$

where x_j is the result for the individual gauge block j and $\overline{x}_j$ the mean over all laboratories for that gauge block. N = 10 is the number of gauge blocks for each material. By the same way the standard deviations s of the differences x_j - $\overline{x}_j$ has been calculated:

$$s^2 = \frac{1}{N-1}\sum_{j=1}^{N}\left((x_j - \overline{x}_j) - < \Delta l >\right)^2 \qquad (2)$$

The normalised standard deviation s_n is the standard deviation divided by the standard uncertainty u_j given by the laboratory for the individual gauge block j

$$s_n{}^2 = \frac{1}{N-1}\sum_{j=1}^{N}\left(\frac{(x_j - \overline{x}_j - < \Delta l >}{u_j}\right)^2 \qquad (3)$$

In table 9, the three figures described above are given for each laboratory and gauge block material separately.

	Steel gauge blocks			**Tungsten carbide gauge blocks**		
	<Δ*l*> / nm	*s* / nm	s_n	<Δ*l*> / nm	*s* / nm	s_n
OFMET	-1.2	6.8	0.6	2.1	5.6	0.5
NPL	-2.8	11.3	0.4	1.5	7.6	0.4
LNE	-2.0	5.0	0.5	14.9	3.0	0.3
NRC	-8.9	9.5	0.5	-0.9	7.9	0.4
NIST	2.9	5.8	0.5	8.6	5.1	0.6
CENAM	-7.1	3.6	0.6	-16.8	6.0	0.9
VNIIM	8.4	16.0	1.8	-28.0	12.9	1.2
CSIRO	6.1	10.0	1.1	14.9	7.2	0.8
NRLM	3.1	14.5	1.1	-2.2	8.8	1.0
KRISS	-11.1	12.1	1.0	-2.2	8.3	0.7
NIM	13.7	9.1	0.8	8.1	5.8	0.7

Table 9. Average deviation, standard deviation from mean and normalised standard deviation from mean for all laboratories.

9.2 Statistical distribution

The figures 7 (a) and (b) show the histogram of the deviations from the mean of all measurement results, for steel and tungsten carbide separately. The standard deviations for all differences from the mean are remarkably small, with s = 12 nm for steel and s = 11 nm for tungsten carbide.

For the histogram of the tungsten carbide gauge blocks, the results of one laboratory (VNIIM) have been excluded. These results for tungsten carbide gauge blocks wrung to quartz flats are without the necessary phase correction and were considered to belong to another measurand, a fact which is underlined by an average deviation of –30.8 nm with respect to the mean of the other laboratories (see section 5).

The representation of all measurement results, independent of their associated uncertainty, in the same histogram (as shown in figure 7) is questionable, because the long gauge blocks were calibrated with considerably larger measurement uncertainties. Therefore, also the normalised differences from the mean (each difference was divided by the standard uncertainty given for the individual result) are represented in a histogram (figures 8 (a) and (b)). Again, the results of VNIIM were excluded.

Both histograms in figure 8 look now somewhat smoother and more symmetrical. The standard deviation of the normalised differences is s = 1.1 and s = 1.2 for the two materials, respectively. For a normal distribution, and assuming that the spread of the results is characterised by their associated uncertainties, s should ideally be 1.

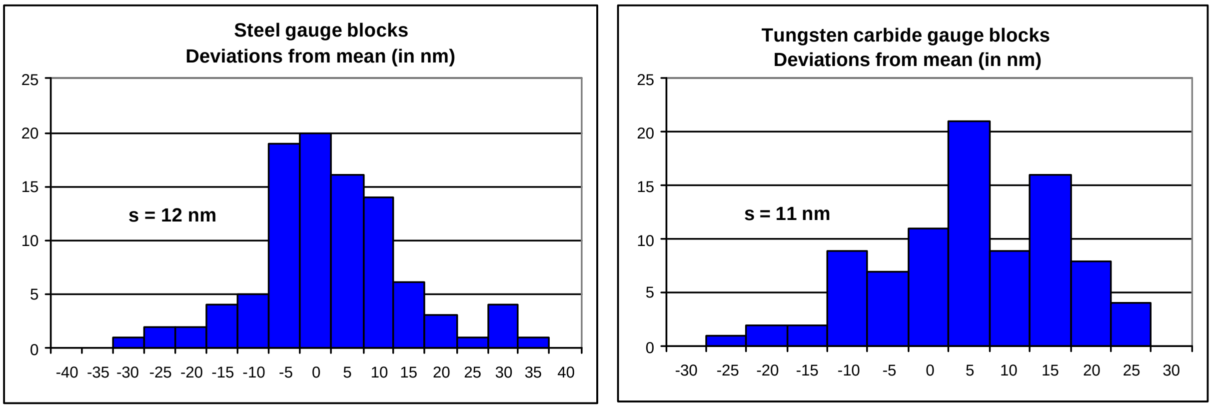


Figure 7(a) and (b). Deviations from mean for all gauge blocks and all laboratories.

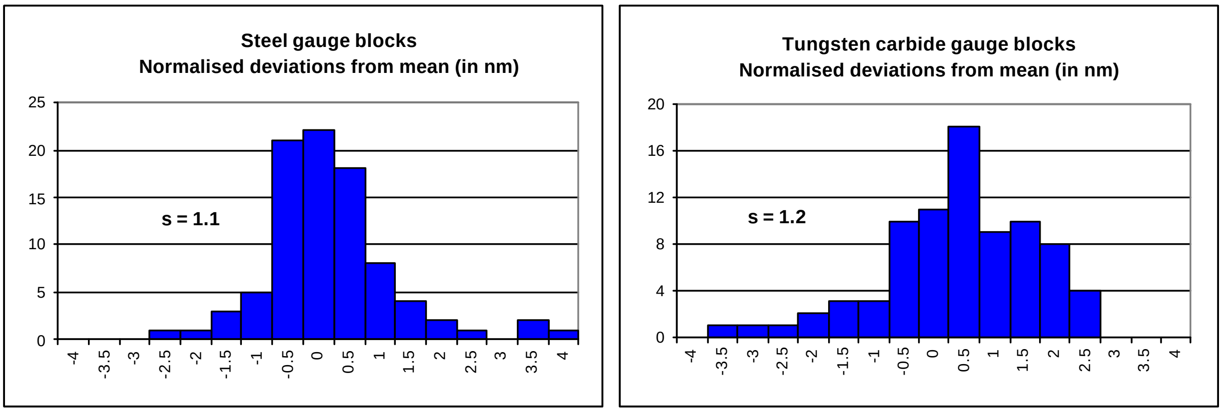


Figure 8 (a) and (b). Histograms of deviations normalised by their associated standard uncertainties.

9.3 Statistical consistency

The statistical consistency of a comparison can be investigated by the so-called Birge ratio R_B [4], which compares the observed spread of the results with the spread expected from the individual reported uncertainties.

The application of least squares algorithms and the χ^2-test leads to the Birge ratio

$$R_B = \frac{u_{ext}}{u_{in}}, \tag{4}$$

where u_{in}, the internal standard deviation, is given by the reported uncertainties

$$u_{in} = \left(\sum_{i=1}^{n} u^{-2}(x_i) \right)^{-1/2}, \tag{5}$$

and the external standard deviation u_{ext} is the standard deviation of the spread of the results x_i, weighted by the associated uncertainties $u(x_i)$:

$$u_{ext} = \left(\frac{\sum u^{-2}(x_i)(x_i - \bar{x}_w)^2}{(n-1)\sum u^{-2}(x_i)} \right)^{1/2}. \tag{6}$$

$\bar{x}_w$ is the weighted mean given by

$$\bar{x}_w = \frac{\sum_{i=1}^{n} u^{-2}(x_i) \cdot x_i}{\sum_{i=1}^{n} u^{-2}(x_i)}. \tag{7}$$

The Birge ratio has an expectation value of $R_B = 1$. For a coverage factor of $k = 2$, the data in a comparison are consistent provided that

$$R_B < \sqrt{1 + \sqrt{8/(n-1)}}. \tag{8}$$

For $n = 11$ participating laboratories, a value $R_B > 1.38$ implies that the laboratories have underestimated their uncertainties.

Table 10 shows the Birge ratio calculated according to Eqs. (4, 5 and 6) for steel and tungsten carbide gauge blocks (for TC without VNIIM results, see section 5):

Steel		**TC**	
L_{nom}	R_B	L_{nom}	R_B
0.5	0.74	**0.5**	1.56
1.01	1.67	**1**	1.51
6	1.10	**1.01**	1.09
7	0.93	**1.1**	1.54
8	1.52	**6**	1.13
15	1.34	**7**	1.29
80	0.79	**8**	1.22
90	1.32	**80**	0.92
100	0.82	**100**	1.26

Table 10. Birge ratio calculated for reported measurement results.

10 Conclusions

From the CCL gauge block key comparison, the following conclusions can be drawn:

- It took two years from the decision to carry out this comparison until the first draft report was available. This can be regarded as a very short time and was only possible thanks to the excellent performance of the participating laboratories in keeping to the original time schedule.
- The decision to limit the number of participating laboratories to about a dozen was justified as the surface quality of the steel gauge blocks at the end of the comparison was poor. Additional measurements could have seriously degraded the measurement results.
- The incorrect results reported by some laboratories for the rogue standard (90 mm tungsten carbide gauge block 1.391 mm out of tolerance), underlines the necessity for a mechanical comparison before the interferometric calibration, where no results of previous measurements are available. It has however, to be noted, that some participants are not equipped for mechanical comparison measurements and would usually subcontract such measurements to an accredited laboratory. It is obvious, that this might have caused problems within the very limited time allocated to each laboratory. Therefore, these results were withdrawn from the comparison.
- The measurement results of most of the laboratories are in good agreement with each other. The standard deviation from the average is for most of the laboratories smaller than the stated standard uncertainties. The standard deviation for the tungsten carbide gauge blocks was generally smaller than that for steel. This is certainly due to the excellent surface quality and the good wringing capability of the tungsten carbide gauge blocks.
- The standard deviation does however, not take into account any systematic offset. For the tungsten carbide gauge blocks some laboratories show a significant offset with respect to the arithmetic mean value, most likely caused by the applied phase correction. These laboratories used platens for wringing the gauge block that were made of a different material from tungsten carbide.
- Two laboratories did not apply any phase correction. It must be outlined that the application of such corrections, prescribed in the international standards, is most important, not only to take into account any differences in the material, but also the different surface roughness of the gauge blocks and the platens.
- It is interesting to note, that of the four laboratories that used the same make of interferometer (nearly duplicate systems), these labs nearly spanned the full range of results and reported uncertainties. This shows that the measurement capability does not only depend on the instrument, but also on other factors such as wringing skill, phase correction, quality of reference platen and calibration of the instruments used to measure the influence quantities, in particular temperature and pressure.
- The WGDM proposed the following facts to be considered for a next CCL gauge block comparison: ceramic should be included as an additional material, the use of rogue gauges should be avoided, reference flats shall not be included in the com-

parison package, the expansion coefficient of the gauge blocks shall not be given with an exceptionally small uncertainty, but only as a manufacturer specification (to be verified by the pilot laboratory).

11 References

[1] T.J. Quinn, Guidelines for key comparisons carried out by Consultative Committees, draft of 21. November 1997, BIPM, Paris. In the meantime, this document has been replaced by a new version: *Guidelines for CIPM key comparisons*, 1. March 1999 (available at http://www.bipm.fr).

[2] This 90 mm gauge block has kindly been loaned by Han Haitjema, Technical University Eindhoven, Netherlands.

[3] J.E. Decker and J.R. Pekelsky, Uncertainty Evaluation for the Measurement of Gauge Blocks by Optical Interferometry, *Metrologia* **34,** 479-493 (1997).

[4] *Statistical Analysis of Interlaboratory Comparisons*, EUROMET workshop held at NPL on 11.-12. November 1999, http://www.npl.co.uk/ssfm/download/documents/sss_m_00_173.pdf

Appendix 1: Measurement conditions

Lab.	Make and type of interferometer	Light sources, Wave lengths	Fringe fractioning method	Refractive index determination	Temperature range / °C
OFMET	NPL TESA, Twyman Green	HeNe Laser TESA, 633 nm HeNe Laser TESA, 543 nm	Sinusoidal interpolation of vertical scans through fringe pattern in video image	Edlén	
NPL	NPL TESA, Twyman Green	HeNe Laser TESA, 633 nm HeNe Laser TESA, 543 nm	Sinusoidal interpolation of vertical scans through fringe pattern in video image	Edlén	19.94 - 20.11
LNE	NPL TESA, Twyman Green	HeNe Laser HP, 633 nm Nd:YAG Laser BNM/INM 532 nm	Using BNM/LNE analysis software in video image	Edlén	19.8 - 20.2
NRC	NRC, Twyman Green	HeNe Laser Coherent, 633 nm HeNe Laser TESA, 612 nm HeNe Laser TESA, 543 nm	Localisation by eye to fiducial in video image	Edlén	19.98 - 20.02
NIST	NPL Hilger Watts, Twyman Green	HeNe Laser Spectra Physics, 633 nm	Visual interpolation of live fringe pattern	Edlén	20.15 - 20.25
CENAM	NPL TESA, Twyman Green	HeNe Lasers TESA, 633 nm and 543 nm	Sinusoidal interpolation of vertical scans through fringe pattern in video image	Edlén	20 ± 0.25
VNIIM	VNIIM Gauge block interf.	HeNe Laser, 633 nm ^{114}Cd spectral lamp	Combined laser and wight light interferometer	Interference refractometer	20 ± 0.1
CSIRO	Hilger Watts, Twyman Green	I_2-stab. HeNe Laser, 633 nm I_2-stab. HeNe Laser, 612 nm I_2-stab. HeNe Laser, 543 nm	Manual selection of fringe position, automatic determination of fringe fraction	Edlén	19.87 - 20.05
NRLM	NRLM Tsugami, Twyman Green	I_2-stab. HeNe Laser, 633 nm ^{198}Hg spectral lamp	Manual positioning of the fringe between reticles by moving optical wedge	Edlén	19.9 - 20.4
KRISS	Tsugami, Twyman Green	Cd spectral lamp Hg spectral lamp	Manual positioning of the fringe between reticles by moving optical wedge, reading taken from linear gauge attached to wedge	Edlén	20 - 21
NIM	Koesters, Carl Zeiss Jena	Lamb dip stab. HeNe Laser, 633 nm	Computer assisted interpolation in video image	Edlén	19.9 - 20.2

Table A1. Measurement instruments and conditions reported by the participating laboratories.

Appendix 2: Determination of the reference values

Proposals for key comparison reference values are worked out according to two different methods, i.e. the arithmetic mean and the weighted mean.

Note that the results of the pilot laboratory contribute only once to the calculation of the reference values, namely by the first measurement, because it has to be assumed, that the further measurements are correlated to some extent.

A2.1 Arithmetic mean

The arithmetic mean reference value x_{ref} is calculated by the average of all measurement values x_i:

$$x_{ref} = \frac{1}{n}\sum_{i=1}^{n} x_i \ . \qquad \text{(A1)}$$

The arithmetic mean does not take into account the uncertainty of the individual results contributing to the reference value. For a relatively small number of participants, results with large deviations, but still not to be considered as outliers, can strongly influence the mean.

The standard uncertainty $u(x_{ref})$ of the arithmetic mean can either be determined by application of the error propagation law, i.e. by taking into account the uncertainties $u(x_i)$ of the individual results [Eq. (A2a)], or by the spread of the results, i.e. by the standard deviation divided by the square root of the number n of results contributing to the mean [Eq. (A2b)].

$$u(x_{ref}) = \frac{1}{n}\sqrt{\sum_{i=1}^{n} u^2(x_i)} = \frac{u_{rms}}{\sqrt{n}} \text{ , or} \qquad \text{(A2a)}$$

$$u(x_{ref}) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - x_{ref})^2} \ . \qquad \text{(A2b)}$$

A2.2 Weighted mean

The weighted mean reference value x_{ref} is calculated by the mean of all measurement values x_i weighted by the inverse square of the standard uncertainties $u(x_i)$ associated with the measurements.

$$x_{ref} = \frac{\sum_{i=1}^{n} u^{-2}(x_i)\cdot x_i}{\sum_{i=1}^{n} u^{-2}(x_i)} \qquad \text{(A3)}$$

The weighted mean approach requires the individual uncertainties from the laboratories be estimated according to a common approach (which should be the case, since all participants were requested to estimate the uncertainties according to the *ISO Guide*). If this is not the case, a single "wrong" value with a strongly underestimated (too small) uncertainty could strongly influence or even fully determine the weighted mean. On the other hand, a high quality measurement with overestimated uncertainty would contribute to the reference value only to a small extent.

The standard uncertainty $u(x_{ref})$ of the reference value is calculated either by appropriately combining the individual uncertainties [Eq. (A4a)], or by the spread of the results [Eq. (A4b)], which is identical to the external standard deviation given in Eq. (6) of section 9.3).

$$u(x_{ref}) = \left(\sum_{i=1}^{n} u^{-2}(x_i) \right)^{-1/2} \qquad \text{(A4a)}$$

$$u(x_{ref}) = \sqrt{\frac{\sum u^{-2}(x_i)(x_i - x_{ref})^2}{(n-1)\sum u^{-2}(x_i)}} \,. \qquad \text{(A4b)}$$

It has to be noted that Eqs. (A2b) and (A4b) do not result from the law of error propagation and are certainly not in accordance to the GUM. In statistically consistent cases, these standard deviations should be approximately equal to the standard uncertainties evaluated according to Eqs. (A2a) and (A4a), respectively, resulting in a Birge ratio of approximately 1 (see section 9.3).

A2.3 Uncertainty of the difference between results and the reference values

For calculating the uncertainty of the difference between an individual result and the reference value, the corresponding uncertainties $u(x_i)$ and $u(x_{ref})$ cannot simply be geometrically added, because the values x_i and x_{ref} are correlated. It can be shown[1], that for the weighted mean with an uncertainty $u(x_{ref})$ according to Eq.(A4a), the expanded uncertainty $U(\Delta x)$ of the difference $\Delta x = x_i - x_{ref}$ is given by

$$U(\Delta x) = 2\sqrt{u^2(x_i) - u^2(x_{ref})} \,. \qquad \text{(A5)}$$

For the arithmetic mean approach, the expanded uncertainty $U(\Delta x)$ is given by

$$U(\Delta x) = 2\sqrt{\left(1 - \frac{2}{n}\right) u^2(x_i) + \frac{1}{n^2} \sum_{j=1}^{n} u^2(x_j)} \,. \qquad \text{(A6)}$$

In the case, where all uncertainties $u(x_i)$ are equal, this results in an expression identical to Eq.(A5).

If external standard deviations are taken for the uncertainty of $u(x_{ref})$, according to Eqs. (A2b) and (A4b), there is no way to calculate the expanded uncertainty $U(\Delta x)$ analytically by application of the law of error propagation. The assumption, that $U(\Delta x)$ be given by the geometric sum of $u(x_i)$ and $u(x_{ref})$ would, however, lead to a too large value, since correlation between x_i and x_{ref} are not taken into account.

A2.4 Exclusion of results contributing to the reference values

Before calculating the reference values, it must be assured, that there are no “outliers” or erroneous results which may significantly bias the reference value. Looking

---

[1] see e.g. Lars Nielsen, *Evaluation of measurement intercomparisons by the method of least squares*, paper presented at [4].

at the graphical representations in figures 4 and 5 and the histograms in figure 8, no single value can be identified to be clearly outlying.

The results of VNIIM for the tungsten carbide gauge blocks show an average deviation of –30.8 nm with respect to the mean of the other laboratories. The physical reason is the phase correction, which has not been applied, although quartz platens were used to wring the tungsten carbide gauge blocks.

The CCL Working Group Dimensional Metrology (WGDM) pointed out, that this procedure did not comply with the technical protocol and the requirements of the international standard ISO 3650 and that a non-corrected value represents a different measurand. According to chapter 5, phase corrections were also not applied by VNIIM for steel and by NIM for both, steel and tungsten carbide, although with a much smaller effect on the results, since the material of the platens used was the same. The WGDM therefore decided, that these laboratories were consequently to be excluded from the determination of the reference value for both materials.

A2.5 Reference values and comparison of the different methods

In table A2, the reference values and their associated standard uncertainties for both, the arithmetic (non-weighted) and the weighted mean method are summarized. The uncertainties of the reference values were calculated according to Eqs. (A2a) and (A4a), respectively; the uncertainty of the difference has been calculated from the geometric sum of the standard uncertainties. The differences between the arithmetic and the weighted mean compared to their uncertainties do not appear to be significant.

Steel gauge blocks				Tungsten Carbide gauge blocks			
Nom	**A.Mean**	**W.Mean**	**Difference**	**Nom**	**A.Mean**	**W.Mean**	**Difference**
0.5	21.4 ± 3.5	20.9 ± 3.2	**0.5 ± 4.7**	**0.5**	24.2 ± 3.4	21.5 ± 3.1	**2.6 ± 4.6**
1.01	28.7 ± 3.5	29.2 ± 3.3	**-0.5 ± 4.8**	**1**	16.4 ± 3.4	14.2 ± 3.1	**2.2 ± 4.6**
6	46.6 ± 3.6	48.7 ± 3.3	**-2.0 ± 4.9**	**1.01**	27.6 ± 3.4	26.5 ± 3.1	**1.1 ± 4.6**
7	29.6 ± 3.5	29.5 ± 3.2	**0.1 ± 4.7**	**1.1**	-51.4 ± 3.4	-52.9 ± 3.2	**1.5 ± 4.7**
8	-0.2 ± 3.5	0.1 ± 3.2	**-0.3 ± 4.8**	**6**	-46.6 ± 3.4	-47.8 ± 3.1	**1.2 ± 4.6**
15	21.1 ± 3.7	21.8 ± 3.3	**-0.7 ± 5.0**	**7**	30.2 ± 3.4	28.8 ± 3.1	**1.5 ± 4.6**
80	28.5 ± 5.8	28.5 ± 5.1	**-0.1 ± 7.7**	**8**	48.7 ± 3.5	47.1 ± 3.1	**1.6 ± 4.7**
90	-18.6 ± 6.3	-17.4 ± 5.5	**-1.2 ± 8.3**	**80**	104.4 ± 4.6	103.9 ± 4.0	**0.5± 6.1**
100	-112.9 ± 6.8	-108.1 ± 5.9	**-4.8 ± 9.0**	**100**	-78.8 ± 5.1	-76.4 ± 4.3	**-2.4 ± 6.7**

Table A2. Reference values and associated standard uncertainties (in nm) calculated according to the arithmetic mean and the weighted mean for all gauge blocks, as well as differences between the arithmetic and the weighted mean values with standard uncertainties of these differences (in nm).

For the choice of one or the other approach, the following facts may be taken into account: The differences between the measurement uncertainties stated by the laboratories are relatively small. This can be explained by the common approach for the estimation of the measurement uncertainty which was given in the technical protocols of this comparison. In addition, the measurement method and the equipment used by the participants are all quite similar. Therefore the remaining differences in the stated uncertainties are partly due to a more or less conservative judgement of the uncertainty contributions from the influence quantities. It is therefore felt that weighing the results by their uncertainty for the determination of the reference values is not appropriate. For the evaluation of the degree of equivalence, the **arithmetic mean reference values** are proposed.

Appendix 3: Comparison with reference value

Tables A3 (a) and (b) show the differences Δl of measured lengths with respect to the arithmetic mean reference values and the expanded uncertainties $U(\Delta x)$ of these differences calculated by Eq.(A6).

	0.5	**1.01**	**6**	**7**	**8**	**15**	**80**	**90**	**100**
OFMET	-4 ± 17	5 ± 17	5 ± 16	1 ± 16	-1 ± 16	-5 ± 16	-6 ± 23	-2 ± 25	17 ± 27
NPL	-1 ± 26	-3 ± 26	8 ± 26	4 ± 26	2 ± 26	1 ± 27	10 ± 51	5 ± 56	-27 ± 60
LNE	-6 ± 19	-4 ± 19	7 ± 19	5 ± 19	4 ± 19	-1 ± 19	0 ± 27	-5 ± 29	3 ± 31
NRC	8 ± 24	-1 ± 24	-11 ± 26	0 ± 26	2 ± 26	-7 ± 26	-19 ± 39	-18 ± 41	-13 ± 44
NIST	5 ± 17	13 ± 17	10 ± 18	4 ± 18	9 ± 18	9 ± 20	5 ± 31	-4 ± 33	-4 ± 34
CENAM	-6 ± 14	-9 ± 14	0 ± 14	-4 ± 14	-3 ± 15	-8 ± 15	-7 ± 30	0 ± 33	-6 ± 36
VNIIM		31 ± 17	21 ± 18	-5 ± 17	32 ± 17	15 ± 18	-3 ± 27	-13 ± 31	9 ± 33
CSIRO	7 ± 17	17 ± 17	6 ± 17	7 ± 17	12 ± 17	30 ± 18	-1 ± 27	-1 ± 29	-1 ± 31
NRLM	3 ± 17	-11 ± 19	-3 ± 20	-3 ± 17	-2 ± 19	-6 ± 21	19 ± 26	28 ± 28	23 ± 32
KRISS	-3 ± 24	-8 ± 23	-25 ± 25	-17 ± 21	-24 ± 21	-13 ± 24	2 ± 32	0 ± 36	9 ± 39
NIM	9 ± 13	19 ± 13	9 ± 13	12 ± 13	12 ± 13	7 ± 13	16 ± 21	37 ± 23	23 ± 25

Table A3(a). Differences of measured lengths of steel gauge blocks with respect to the arithmetic mean reference values and expanded uncertainties of these differences.

	0.5	**1**	**1.01**	**1.1**	**6**	**7**	**8**	**80**	**100**
OFMET	-1 ± 17	-1 ± 17	-4 ± 17	-3 ± 17	-3 ± 16	-4 ± 16	-4 ± 16	7 ± 20	13 ± 22
NPL	0 ± 26	-1 ± 26	-5 ± 26	0 ± 26	-1 ± 26	-1 ± 26	5 ± 26	12 ± 31	-15 ± 35
LNE	17 ± 19	14 ± 19	9 ± 19	15 ± 19	13 ± 19	14 ± 19	7 ± 19	12 ± 21	14 ± 22
NRC	5 ± 24	2 ± 24	1 ± 24	0 ± 24	-1 ± 26	-2 ± 26	0 ± 26	-14 ± 38	-18 ± 44
NIST	11 ± 17	8 ± 17	4 ± 17	13 ± 17	6 ± 18	9 ± 18	9 ± 18	-3 ± 30	1 ± 33
CENAM	-26 ± 14	-25 ± 14	-18 ± 14	-21 ± 14	-14 ± 14	-13 ± 14	-14 ± 14	-17 ± 19	-22 ± 21
VNIIM	0 ± 17	-25 ± 17	-40 ± 17	-31 ± 17	-49 ± 17	-36 ± 17	-19 ± 17	-40 ± 26	-31 ± 30
CSIRO	7 ± 17	17 ± 17	11 ± 17	19 ± 17	18 ± 17	19 ± 17	15 ± 17	1 ± 21	8 ± 23
NRLM	-5 ± 17	-4 ± 17	7 ± 17	-15 ± 19	-4 ± 17	-17 ± 17	-8 ± 17	-3 ± 20	9 ± 21
KRISS	-6 ± 19	-8 ± 19	-8 ± 18	-11 ± 18	-11 ± 19	-3 ± 20	-11 ± 20	8 ± 27	11 ± 30
NIM	8 ± 13	5 ± 13	0 ± 13	1 ± 13	-1 ± 13	2 ± 13	9 ± 13	9 ± 17	21 ± 19

Table A3(b). Differences of measured lengths of tungsten carbide gauge blocks with respect to the arithmetic mean reference values and expanded uncertainties of these differences.

Appendix 4: Transfer of reference values to RMO key comparisons

In order to appropriately link CCL with RMO key comparisons in the case of artefact based comparisons, the application of the concept to transfer the key comparison reference value to a second, independent comparison, turns out to be difficult. Not only that the reference values do not have the importance of a realisation of an SI unit (they have no metrological significance apart from the fact that they represent the best estimate of a particular measurand of an artefact at a given time), but also a rigorous transfer of the reference value would necessitate the introduction of metrologically meaningless corrections and lead to an undue increase in the uncertainty of the regional reference value used to express the degree of equivalence.

An alternative more adequate approach is the one which has been applied by the CCEM-K4 comparison where the key comparison reference value is the nominal value of the standard. This reflects the fact, that for external parties the exact value of the measurand (e.g. the length of the gauge blocks) has no further meaning than their nominal length, except for the purpose of expressing the equivalence. It complies also with the common practice of expressing calibration results for material standards as the deviation from nominal value.

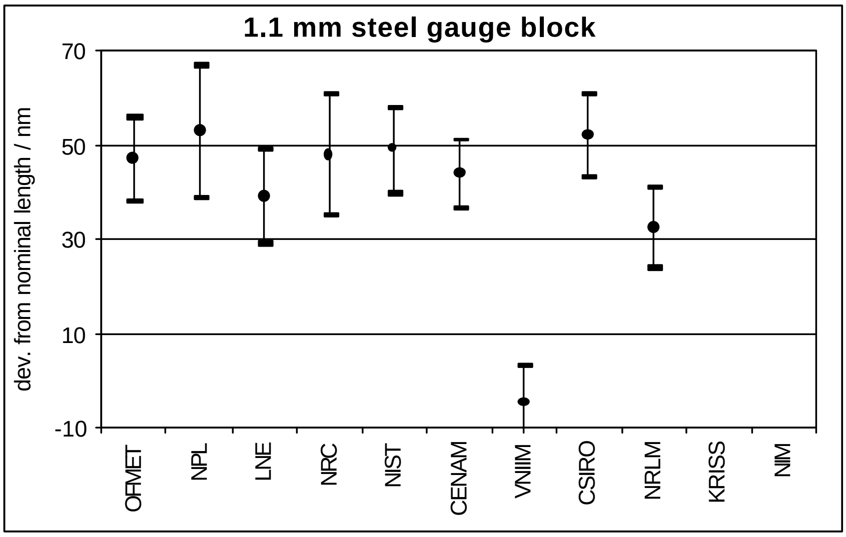
Practically this is done in the following way:
For determining the degree of equivalence with respect to the reference value, the arithmetic (or weighted) mean x_a and the uncertainty $u(x_a)$ of this mean are calculated. The degree of equivalence is given by the difference $D_i = (x_i - x_a)$ and the expanded uncertainty $U(D_i)$ of this difference. The key comparison reference value x_R is obtained from the arithmetic mean x_a of the participants values x_i by adding a constant C chosen such that the reference value is the nominal length L: $x_R = x_a + C = L$. C is thus the deviation of x_a from nominal length.The transfer of the key comparison reference value to a regional comparison is straight forward if similar artefacts of the same quality with the same nominal values are used in the regional comparison. Since the nominal value has no uncertainty, there is no additional uncertainty contribution to be taken into account in the regional comparison when calculating the degree of equivalence with respect to the reference value.

The calculation of the mutual degree of equivalence between two laboratories is not recommended for comparisons involving several material standards, since it cannot be expressed in a single pair of values. It would be given by $D_{ij} = (x_i - x_j)$ and the expanded uncertainty $U(D_{ij})$ of this difference for two laboratories participating in the same comparison, and by $D_{ij} = (D_i - D_j)$ and its expanded uncertainty $U(D_{ij})$ for two laboratories participating in distinct comparisons.

The proper link between two comparisons is established by an expert judgment of the results of the participants common to both comparisons, taking into account their degrees of equivalence for all standards of the two comparisons.

Appendix 5: Results not to be published

During the comparison the measurement surfaces of the 1.1 mm steel gauge block were severely scratched, such that wringing became impossible towards the end of the circulation. Therefore, some laboratories did not report any result and comparability of the reported results (see graph below) cannot be guaranteed.



As mentioned in the main part of this report, the 90 mm tungsten carbide gauge block was 1.39145 mm shorter than its nominal length. It has been manufactured in such a length as to have similar excess fringe fractions as a 90 mm gauge block for all laser and spectral lamp wavelengths usually used in gauge block interferometry. Only laboratories making a predetermination of the length by mechanical comparison could become aware of this fact. These results do therefore not necessarily represent the measurement capability of the laboratories. The graph below shows the results of those laboratories having perfomed a mechanical comparison before the interferometric calibration.